Abstract

It is known that signals (which could be functions of continuous space or time), belonging to $L_2$-space, cannot be localized simultaneously in space/time and frequency domains. Alternatively, signals have an effective space/time-bandwidth product (SBP) with a positive lower bound. This is the uncertainty inequality (UI), established by Gabor [1] who also showed that the Gaussian function is the only signal that has the lowest SBP.

A continuous-time bandlimited signal in $L_2$ obeys the Shannon sampling theorem and, at the same time, allows an interpolation scheme for exactly reconstructing it from its discrete samples. We analyze the relationship between the concentrations in the temporal and spectral domains of (i) continuous-time; and (ii) discrete-time signals. The former is governed by the Gabor uncertainty inequality, while its natural discrete counterpart seems to exhibit some strange properties. Since the discrete version of a bandlimited signal is an exact representation of its continuous version, we expect the effective spreads (numerically) of the signal in both the continuous and discrete-time cases to be the same.

We show that some of the existing definitions of effective spreads for discrete-time this discrepancy are not consistent. We deal with the following problem: Suppose we are given appropriately sampled values of a bandlimited signal. Can we express the effective spatial and spectral spreads in terms of these samples, using the standard definitions of the spatial and spectral spreads meant for the continuous version of the signal, such that there is consistency in the definitions in both the continuous and discrete-time cases? In contrast with the results of the literature, we present a new approach to solve this problem. We also present a comparison of the results obtained using the proposed definitions with those available in the literature.

Since the Gaussian function is not bandlimited, no bandlimited signal can have the lowest SBP. This motivates us to formulate the following problem: What is the greatest lower limit (*infinimum*) for the SBP of bandlimited continuous-time signals, and how close is this limit to the optimal value specified by UI?

We show that by choosing the convolution product of a Gaussian signal (with $\sigma$ as the variance parameter) and a bandlimited filter with a continuous spectrum, there exists a finite-energy, bandlimited signal whose SBP can be made arbitrarily close (dependent on the choice of
σ) to the optimal value specified by the UI. Also, numerical simulation results are presented to demonstrate that the SBP of this signal can be controlled by varying the σ parameter.

Finally, we propose a technique for designing finite-length discrete signals which have the minimal SBP.

The main contributions of the thesis are:

- A novel approach to compute the effective spreads of a continuous-time bandlimited signal from its samples. The approach is based on a refinement of the standard sinc-function (used for interpolation in the sampling theorem) which facilitates the computation of the effective spatial width.

- An analytical proof which shows that there exists a bandlimited signal whose SBP can be made arbitrarily close to the lower bound specified by the UI.

- A technique to design finite-length discrete signals which have minimal SBP.

The thesis is organized as follows:

- Chapter 1: Importance of bandlimited signals and the uncertainty inequality. Brief review of the relevant literature and motivation for the current work.

- Chapter 2: Gabor inequality of a bandlimited signal in terms of its samples.

- Chapter 3: Determination of optimal bandlimited signals.

- Chapter 4: Conclusions.