Abstract

A discrete time signal $x[n]$ is a sequence of numbers (real/complex) defined with an integer index $n$. Any practical Discrete Time System (DTS) can only recognize a finite set of numbers (quantized), each defined by a fixed number of bits. Hence irrespective of the nature of the operations (linear), the numbers within a DTS are limited by a pre-defined maximum value (dynamic range) and minimum value (precision). This primary limitation, requires a linear scaling of the form $x'[n] = Gx[n]$ where $G$ is a scaling constant. For a finite number of bits, one has to tradeoff the dynamic range with the precision allowed. Moreover, the extremas of any sequence can only be defined for a fixed length or time duration of observation. Hence scaling is only optimal within a pre-defined length of $x[n]$.

For a finite length sequence $x[n]$, $0 \leq n \leq L_X$, if $|x[n]| \leq x_{\text{MAX}}$(maximum absolute value), a simple scaling procedure which ensures that $|x'[n]| < 1$ is that $G \geq 1/x_{\text{MAX}}$. This scaling procedure is no-doubt simple but not always optimum as it protects the larger values of $x[n]$ by neglecting the smaller values. In signal processing literature, there are many other scaling procedures which choose $G$ more carefully with some computational cost. As there is no single optimal rule for scaling (ad-hoc), any scaled sequence $x'[n]$ which is based on some characteristic properties of $x[n]$ itself can avoid this ambiguity in the scaling process. A DTS which maps an input sequence $x[n]$ to another sequence $y[n]$, has to ensure that the operations are such that at any point of time the intermittent numbers within the course of computation and at the end are also within the allowed limits.

In this work, using some spectral properties of a finite length sequence, a novel framework is proposed which maps $x[n]$ to a set of real signed fractions and few complex scaling constants. As signed fractions (zero dynamic range) require only bits to safeguard their precision, these can be easily used across multiple discrete time systems. These fractions also reveal some spectral characteristics of $x[n]$ which can be used (if required) by any DTS. This novel representation framework gives a fundamental representation format of $x[n]$ which we call as the Line Spectral Frequency (LSF) model (LSF-Model) of a sequence.

The Line Spectral Frequency (LSF) of a causal finite length sequence is a frequency at which the spectrum of the sequence annihilates or the magnitude spectrum has a spectral null. A causal finite length sequence with $(L+1)$ samples having exactly $L$-LSFs is referred, as an Annihilating (AH) sequence. The normalized values of these LSFs turn out be signed...
real fractions with maximum magnitude of \( \left( \frac{1}{2} \right) \).

**Overview of an LSF Model:**

It is well known that the spectrum of a finite length sequence \( x[n] \), \( 0 \leq n \leq L_x \) is defined by the roots of the polynomial \( X(z) = \sum_{n=0}^{L_x} x[n]z^{-n} \). Based on the location of the roots of \( X(z) \), relative to the reference contour \( |z| = 1 \), the sequence \( x[n] \) is decomposed into minimum-phase (MP) \(^1\) sequences of smaller length. Using some properties of a minimum-phase sequence and some delay and shift parameters (model parameters), we observe that a minimum-phase sequence can be mapped to two AH-sequences.

The above set of linear operations on the sequence \( x[n] \) are pictorially summarized by a Spectral Decomposition Structure (SDS). An SDS is basically a structured method involving linear operations to compute the spectrum of \( x[n] \) with the spectra of smaller length MP/AH-sequences. Hence a sequence \( x[n] \) is effectively mapped to a set of AH-sequences (defined by LSFs) and few complex constants with a reference SDS. As this process requires a few model parameters, this alternate representation format is referred as the LSF model of the sequence \( x[n] \). Hence the LSF model obtains the fractions using the spectral characteristics of the sequence and does not simply rely on \( x_{\text{MAX}} \) as described above. For the same sequence \( x[n] \), one can obtain multiple LSF models by varying the model parameters. But for a given set of model parameters the mapping is assured to be unique.

The LSF model is a fundamental decomposition, which can be used for both time and frequency domain processing (analysis/synthesis) of the sequence. We show that the AH-sequences have good properties in time and frequency domain and these can also be exploited by any sequence \( x[n] \) only when it is mapped to its LSF model. We present several properties of the LSF model of a sequence and use this model to obtain a novel canonic filter structure and also a novel non-uniform spectral sampling algorithm.

Using some properties of MP-sequences, we also develop the LSF-Format of a sequence, which maps each sample \( x[n] \) independently into a pair of LSFs. Unlike the LSF-Model, the LSF-Format neither uses any model parameters nor relies on the root structure. The LSF-Format is more appropriate for storage of a sequence in practical discrete systems.

As LSFs are normalized phase values, which do not require any scaling, their properties when represented with fixed precision play a vital role in the practical realization of discrete time systems. These properties can be exploited by any signal processing algorithm which works with phasors and normalized phase values.