Abstract

It is well known that there exist domains $\Omega$ in $\mathbb{C}^n$, $n \geq 2$, such that all holomorphic functions in $\Omega$ continue analytically beyond the boundary. We wish to study this remarkable phenomenon. The first chapter seeks to motivate this theme by offering some well-known extension results on domains in $\mathbb{C}^n$ having many symmetries. One important result, in this regard, is Hartogs’ theorem on the extension of functions holomorphic in a certain neighbourhood of $(\bar{\mathbb{D}} \times \{0\}) \cup (\partial\mathbb{D} \times \mathbb{D})$, $\mathbb{D}$ being the open unit disc in $\mathbb{C}$. To understand the nature of analytic continuation in greater detail, in Chapter 2, we make rigorous the notions of ‘extensions’ and ‘envelopes of holomorphy’ of a domain. For this, we use methods similar to those used in univariate complex analysis to construct the natural domains of definitions of functions like the logarithm. Further, to comprehend the geometry of these abstractly-defined extensions, we again try to deal with some explicit domains in $\mathbb{C}^n$; but this time we allow our domains to have fewer symmetries. The subject of Chapter 3 is a folk result generalizing Hartogs’ theorem to the extension of functions holomorphic in a neighbourhood of $S \cup (\partial\mathbb{D} \times \bar{\mathbb{D}})$, where $S$ is the graph of a $\bar{\mathbb{D}}$-valued function $\Phi$, continuous in $\mathbb{D}$ and holomorphic in $\bar{\mathbb{D}}$. This problem can be posed in higher dimensions and we give its proof in this generality. In Chapter 4, we study Chirka and Rosay’s proof of Chirka’s generalization (in $\mathbb{C}^2$) of the above-mentioned result. Here, $\Phi$ is merely a continuous function from $\bar{\mathbb{D}}$ to itself. Chapter 5 — a departure from our theme of Hartogs-Chirka type of configurations — is a summary of the key ideas behind a ‘non-standard’ proof of the so-called Hartogs phenomenon (i.e., holomorphic functions in any connected neighbourhood of the boundary of a domain $\Omega \in \mathbb{C}^n$, $n \geq 2$, extend to the whole of $\Omega$). This proof, given by Merker and Porten, uses tools from Morse theory to tame the boundary geometry of $\Omega$, hence making it possible to use analytic discs to achieve analytic continuation locally. We return to Chirka’s extension theorem, but this time in higher dimensions, in Chapter 6. We see one possible generalization (by Bharali) of this result involving $\Phi \in \mathfrak{A}$, where $\mathfrak{A}$ is a subclass of $\mathcal{C}(\mathbb{D}; \mathbb{D}^n)$, $n \geq 2$. Finally, in Chapter 7, we consider Hartogs-Chirka type configurations involving graphs of multifunctions given by “Weierstrass pseudopolynomials”. We will consider pseudopolynomials with coefficients belonging to two different subclasses of $\mathcal{C}(\bar{\mathbb{D}}; \mathbb{C})$, and show that functions holomorphic around these new configurations extend holomorphically to $\mathbb{D}^2$. 