Synopsis

Wear and fatigue characteristics of engineering surfaces are dependent on the contact pressures and the resulting subsurface stress distributions in the bodies which are in contact. Real surfaces are rough and hence the contact between two real surfaces can be considered in general as a contact between two curved rough bodies. This thesis analyses the above problem in its simplest form as a plane strain problem of a rigid cylinder of radius \( r \), indenting an elastic half space carrying a regular array of undulations, shape of which can be represented by a simple mathematical function. The problem is transformed into the problem of indentation of an elastic half-space by a system of normally loaded connected rigid punches with base profile represented by the function of separation \( h(x) \) between the two bodies in contact. This is now a Class-A punch indentation problem with incomplete contact at the base of the punch profiles so that the contact pressures are bounded (and equal to zero) at the ends of the regions of contact. Using the method of complex variables, the above problem has been reformulated in terms of a complex potential \( \Phi(z) \), where \( z = x + iy \), \( i = \sqrt{-1} \), to yield \( 2N \) number of non-linear simultaneous equations in \( 2N \) variables (which are the end points of the regions of contact of \( N \) punches) which can be solved by Newton's iterative scheme. The extent of contact regions and the contact pressure distribution on each region emerge from the analysis a posteriori.
Chapter 1 describes the significance of the problem of contact of rough surfaces and reviews some of the early models and methodologies for tackling this problem. The need for a more realistic and tractable method of dealing with the problem has also been highlighted. The method of complex variables has been identified to be one such method.

Since the problem of rough contact can be converted to an equivalent multiple punch indentation problem, Chapter 2 gives relevant details of the complex variable solution to the problem of indentation of an elastic half-space by a system of connected rigid punches.

In Chapter 3 the contact configuration and contact pressure distribution over the contact regions of an elastic half-space carrying uniformly spaced cylindrical asperities indented by a rigid smooth cylinder has been analysed using complex variable method over a range of normal displacements. The approach has been validated by modelling the same problem using boundary element method (BEM) and the results have been compared with those obtained using a priori Hertzian pressure distribution. It has been observed that the contact is non-Hertzian especially when the asperity tip radius is large compared to the radius of the indenting cylinder. The dynamic nature of the asperity contact has also been established.

When two rough bodies are in normal contact the point where the peak equivalent stress is induced is of particular interest to tribologists and surface engineers since the onset of plasticity occurs at this point which in turn strongly influences the mechanism of wear. Indented by a large rigid smooth cylinder, the equivalent shear stress field induced in an elastic half-space carrying equi-spaced cylindrical asperities on its surface,
has been analysed in Chapter 4, for different asperity tip radius-asperity pitch-applied load combinations. For every combination, it has been observed that there exists two peaks of equivalent shear stress, one at the individual asperity (local) and the other, in the subsurface (global). Using complex variable analysis, it has been shown that there exists such combinations for which the global subsurface maximum (equivalent) shear stress is more than the maximum local (equivalent) shear stress at any asperity. Thus, it is possible that the subsurface starts yielding even when the asperities are elastic.

A finite element (FEM) model using a commercial FEM package called ABAQUS has been used to compare and verify the above result.

The process of the development of contact or what happens to the asperities under load is of great engineering importance since (i) the quantitative development of the real area of contact provides information about the dynamics of load support which in turn influences friction and wear and (ii) the ability or otherwise of the asperities to persist under load have important bearings on how and what amount of heat is generated at contact and for electrical contact it determines microscopic air gaps and constriction to the flow of electricity. Hence, in Chapter 5, the approach using complex variable method has been used to obtain a relationship between the applied load and the corresponding contact regions developed for a simple configuration of a half space carrying sinusoidal elastic asperities being indented by a hard cylinder.

It has been shown that a power law relationship exists between the load and contact area and the load power factor varies between 0.5 and 1.0 as the surface topography changes from perfectly smooth to very rough. For realistic engineering topography the global nature of contact is Hertzian. It has also been shown by calculating
the closure of the gap between the middle and the next asperity that closely spaced asperities have a tendency to persist while widely spaced short asperities disappear under modest normal loads

The complex variable method used for the present analyses is seen to produce good results for surfaces of finished engineering products, but has the limitation that it ceases to be valid once plasticity is developed somewhere inside the material