Consider the following classical results from Euclidean harmonic analysis:

(I) A function \( f \in L^2(\mathbb{R}^n) \) admits a factorization \( f(x) = g \ast p_t(x) \) (where \( p_t \) is the heat kernel) if and only if \( f \) extends as an entire function to \( \mathbb{C}^n \) and is square integrable with respect to the measure \( p_{t/2}(y)dx\,dy \). This result characterizes the image of the heat kernel transform (Segal-Bargmann transform) as a class of holomorphic functions.

(II) A function \( f \in L^2(\mathbb{R}) \) admits a holomorphic extension to the strip \( \{x + iy : |y| < t\} \) such that

\[
\sup_{|y| \leq s} \int_{\mathbb{R}} |f(x + iy)|^2 \, dx < \infty \quad \forall s < t
\]

if and only if

\[
e^{s|\xi|} \tilde{f}(\xi) \in L^2(\mathbb{R}) \quad \forall s < t
\]

where \( \tilde{f} \) denotes the Fourier transform of \( f \). This result is due to Paley and Wiener.

While the first problem (I) poses a clear question for other Lie groups (namely, can the image of \( L^2 \) under the heat semigroup be characterized as a class of holomorphic functions square integrable with respect to some weight) the second result may be generalized in different ways. Firstly, the result of Paley-Wiener may be viewed as a characterization of the image of \( L^2 \) under the Poisson semigroup \( e^{-t\Delta^{1/2}} \). Secondly, the condition above may be interpreted as \( e^{\xi(x+iy)} \tilde{f}(\xi) \in L^2 \). Notice that \( e^{\xi(x+iy)} \) gives complexified representations of the real line. These point of views lead to interesting questions on other Lie groups.

In this thesis these questions were looked at for two Lie groups, namely, Euclidean motion groups \( (\mathbb{R}^n \ltimes K, \text{ where } K \subset SO(n), \text{ a closed subgroup}) \) and Heisenberg motion groups \( (\mathbb{H}^n \ltimes K, \text{ where } K \subset U(n)) \). Interesting results were obtained which extends the above classical results. Along with the representation theory of these Lie groups, a key ingredient in the proof of these results is a Gutzmer’s formula (due to Lassalle) on compact Lie groups. For the Heisenberg motion groups, the unitary irreducible representations which occur in the Plancherel theorem have been explicitly realized. This was used in proving a Paley-Wiener type theorem using the complexified representations.