Abstract

The 4-genus of a knot in $S^3$ is an important measure of complexity, related to the unknotting number. A fundamental result used to study the 4-genus and related invariants of homology classes is the Thom conjecture, proved by Kronheimer-Mrowka, and its symplectic extension due to Ozsváth-Szabó, which say that closed symplectic surfaces minimize genus.

In this thesis, we prove a relative version of the symplectic capping theorem. More precisely, suppose $(X,\omega)$ is a symplectic 4-manifold with contact type boundary $\partial X$ and $\Sigma$ is a symplectic surface in $X$ such that $\partial \Sigma$ is a transverse knot in $\partial X$. We show that there is a closed symplectic 4-manifold $Y$ with a closed symplectic submanifold $S$ such that the pair $(X, \Sigma)$ embeds symplectically into $(Y, S)$. This gives a proof of the relative version of Symplectic Thom Conjecture. We use this to study 4-genus of fibered knots in $S^3$.

We also prove a relative version of the sufficiency part of Giroux’s criterion for Stein fillability, namely, we show that a fibered knot whose monodromy is a product of positive Dehn twists bounds a symplectic surface in a Stein filling. We use this to study 4-genus of fibered knots in $S^3$. Using this result, we give a criterion for quasipositive fibered knots to be strongly quasipositive.

Symplectic convexity disc bundles is a useful tool in constructing symplectic fillings of contact manifolds. We show the symplectic convexity of the unit disc bundle in a Hermitian holomorphic line bundle over a Riemann surface.